Track Momentum Error

Abstract: The momentum calculation of tracks in the `myanal analysis` is examined with the Monte Carlo. The estimated momentum error of the fit is emphasized.

Introduction

The spectrometer momentum estimate for individual tracks is the primary estimator for charged particles with momenta greater than about 20 GeV/c, above which the estimate using emulsion information loses accuracy. The estimated accuracy of the momentum measurement is the main focus of this study.

1. Rosie Magnetic Field

The spectrometer analysis magnet "Rosie" was completely mapped with a magnetometer using a grid pattern. The error in the an individual probe measurement is about 0.1% and the error in $\int Bdl$ is better than 0.5%. Although all three spatial components of **B** were measured, on the B_y component is used in the tracking using `myanal`, including the incremental or "swim" fit. The values of B_x along the spectrometer axis (x=y= 0) is shown in Figure 1. The variation in $\int Bdl$ for other zero-angle trajectories is shown in Figure 2. The central value of the field integral is 0.227 GeV/c. The maximum values are found near the pole iron. The uncertainty in $\int Bdl$ is small compared to other factors discussed below.

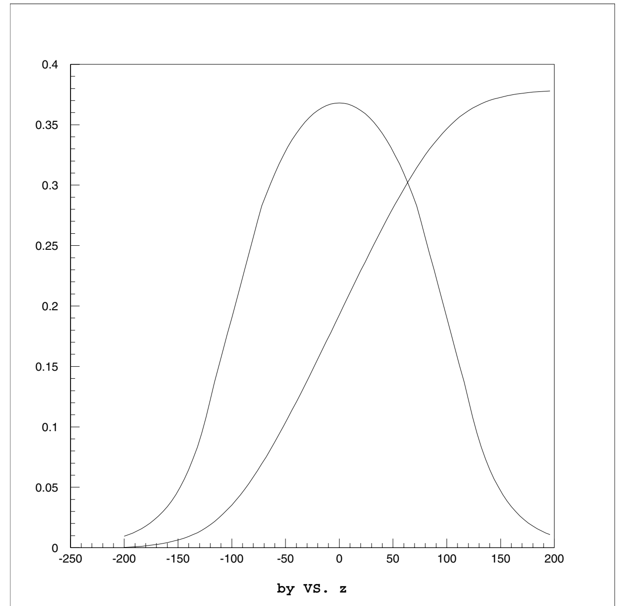

Figure 1. Rosie central field.
The vertical (y) component (in Tesla) and its integral is shown as a function of z (cm),

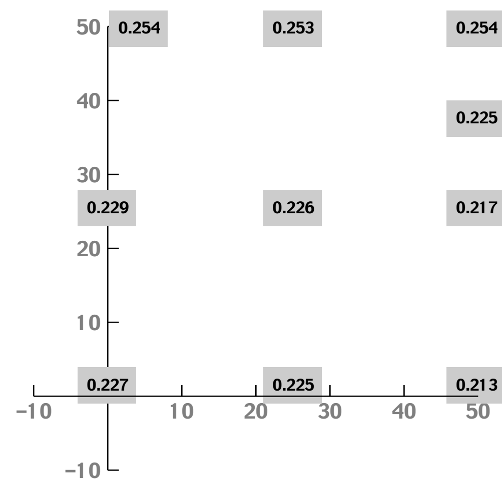

Figure 2. The field integral matrix.
The value is translated to an equivalent transverse momentum kick (GeV/c). The trajectory is straight, with x (horizontal) and y in cm.

2. Track Fitting Procedure

The procedure for fitting tracks in `myanal` is to first get the momentum with a kink fit then if the momentum is estimated to be less than 10 GeV/c a "swim" fit is performed using `ghelix` (with the x and z components ignored). For the kink-fit, a five parameter linear regression is performed with upstream x slope and intercept and downstream x slope (after Rosie) and y slope and intercept (no bend in y assumed). In addition, a vertex constraint is implemented by including it in the fit with a very large weight. The results of this fit using Monte Carlo files of muons of fixed momenta is given in Table 1.

Ideally, the calculated error would be close to the measured error, since the Monte Carlo should reproduce the scattering in material and the resolution effects in each detector. The table shows that the swim fitting ($\leq$ 10 GeV/c) has underestimated errors, and the kink fitted tracks have overestimated errors. The errors in the kink fit are derived from the curvature matrix. The χ^2 function is minimized analytically. Figures 3 - 5 show the distribution of momenta near each central (input) value, Figures 6 - 8 show the inverse momenta ($1/p$) for the momentum bins.

3. Refit with MINUIT

Understanding errors in multi-parameter fitting is usually easier with numerical minimization, even for the linear case such as track fitting to straight lines. The CERN MINUIT package of minimization routines is easily implemented and provides a second method (numerical) for extracting the error in momenta. The error is calculated from

$$\delta\left(\frac{1}{p}\right)^2 \cong \kappa\,\delta(\Delta\theta)^2 = \kappa\left(\delta\theta_{dnst}^2 - \delta\theta_{upst}^2\right)$$

where θ_{dnst} and θ_{upst} are the angles in the x-view after and before the magnet, respectively. They are derived from the fit parameters using $\cos\theta = S_x$ where S is the slope of the line segment. MINUIT calculated parabolic errors about the parameter minima, used in this analysis.

There are numerous approximations to the exact value for the error, which we will now evaluate. First, the track through the magnet is a curve not a kink. The error incurred in the kink approximation is small. For example, assuming a constant $\int Bdl$ for all momenta ignores the additional path length for low momenta particles. The extra length, $\Delta L \sim L/(2\cos\theta)$ exceeds 1% only for $p < 2$ GeV/c. Second, the B_x component is ignored, which would cause the track to kink in y also. This will cause some correlations between the x and y views as seen in the inverse of the 2nd derivative matrix.

Applying the MINUIT fit to 20 GeV/c MC muon tracks, yielded the same values of the parameters as the kink-fitter in `myanal` to 5 decimal places. The χ^2 averages 1.35 per degree-of-freedom with the nominal values of the detector resolutions. Only the SFT and DC data are used in this fit. This indicates that the effective resolution (at least in the MC simulation) is probably too low. Although there are about 1.5 radiation lengths of lead and plastic between these tracking systems, it can only affect the y-view since the two x-segments are independent and outside the scattering material. Additionally, the y-view is weakly defined downstream of the magnet. so it carries little weight in the global fit.

4. Scattering and the Fit

The contribution of scattering to the χ^2 of the fit and error in the momentum can easily be estimated from the amount of material located between the SFT and DCs. P. Berghaus has done the most complete analysis and has derived the relative momentum error based on scattering and chamber resolution :

p GeV/c	μ_p	σ_{dist}	χ^2	$\sigma(p^{-1})_{dist}$	$\delta(p^{-1})_{dist}$	$\delta(p^{-1})_{est}$	$\delta(p^{-1})_{MS}$	$\delta(p^{-1})_{TOT}$
5	5.08	0.69	1.09	0.028	0.0034	0.0040	0.028	0.028
10	10.11	1.13	0.69	0.018	0.0034	0.0040	0.014	0.015
10*	10.20	0.90	0.57	0.013	0.0034	0.0040	0.008	0.009
20	20.8	2.5	0.52	0.012	0.0034	0.0040	0.0069	0.0080
50	48.4	7.4	0.53	0.0035	0.0034	0.0040	0.0028	0.0049
100	103.	21.	0.49	0.0041	0.0034	0.0040	0.0014	0.0042
200	213.	75.**	0.51	0.0036	0.0034	0.0040	0.0007	0.0041

Table 1. Summary of momentum fit with Monte Carlo.
One hundred muon tracks were generated and then reconstructed to give the momenta for each momentum bin. The muons start at $z = 0$ except for a special set at 10 GeV/c that starts at 90 cm (no target scattering).

$$\delta\left(\frac{1}{p}\right) \cong \sqrt{\left(\frac{\delta\vartheta_{MS}}{p_T^{mag}\cdot p}\right)^2 + \left(\delta\vartheta_{meas}\right)^2} = \sqrt{\left(\frac{0.019}{p^2}\right) + 1.6\times 10^{-5}} \quad (1)$$

where the first term in the radical is due to scattering and the second from resolution. The numbers for errors in $1/p$ deduced from fitting the MC events, in Table 1 column 5, are well reproduced by the simple model formula, shown in Table 1 column 9 (last). For the contribution from measurement MINUIT gives a constant 0.0034 (GeV/c)$^{-1}$ while the value from the Berghaus thesis is 0.0043. We assumed a value in-between of 0.0040 which fits the measured (MC of course) values better and implicitly compensates for real-world effects such as missing DC points.

5. Conclusions

It is recommended that for tracks that are well-fit in the spectrometer, both SFT and DC, the form Eq.(1) be adopted to estimate the error in momenta. It can be applied accurately if:

- there are ≥ 8 DC points
- there is no hit assignment confusion in DC or SFT
- the momentum is > 5 GeV/c

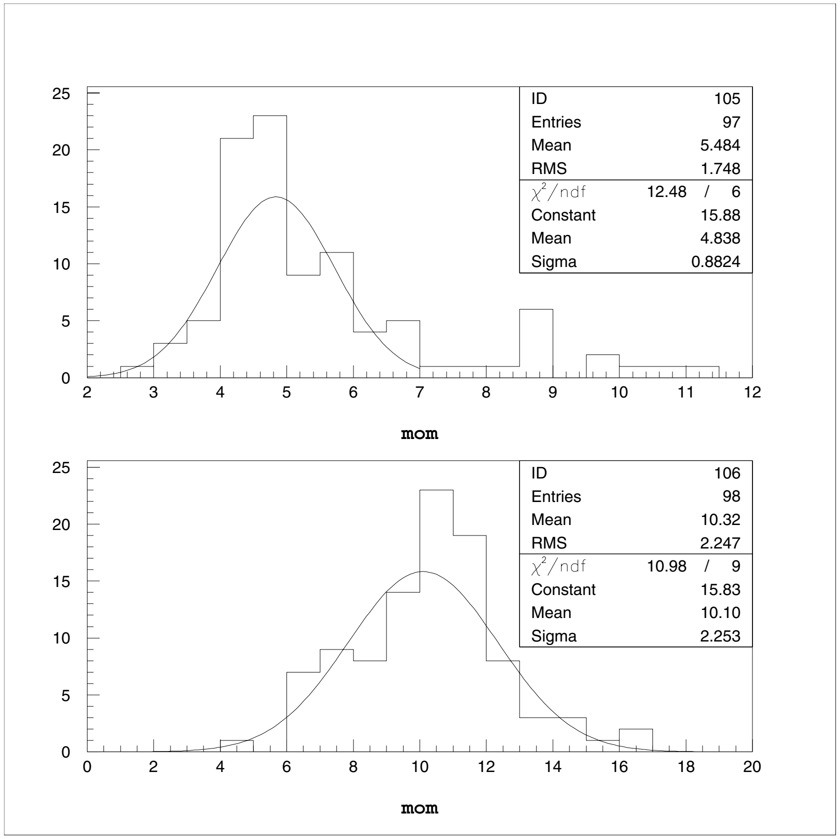

Figure 3. Reconstructed muon track momenta. The figure shows (*top*) 5 GeV/c and (*bottom*) 10 GeV/c.

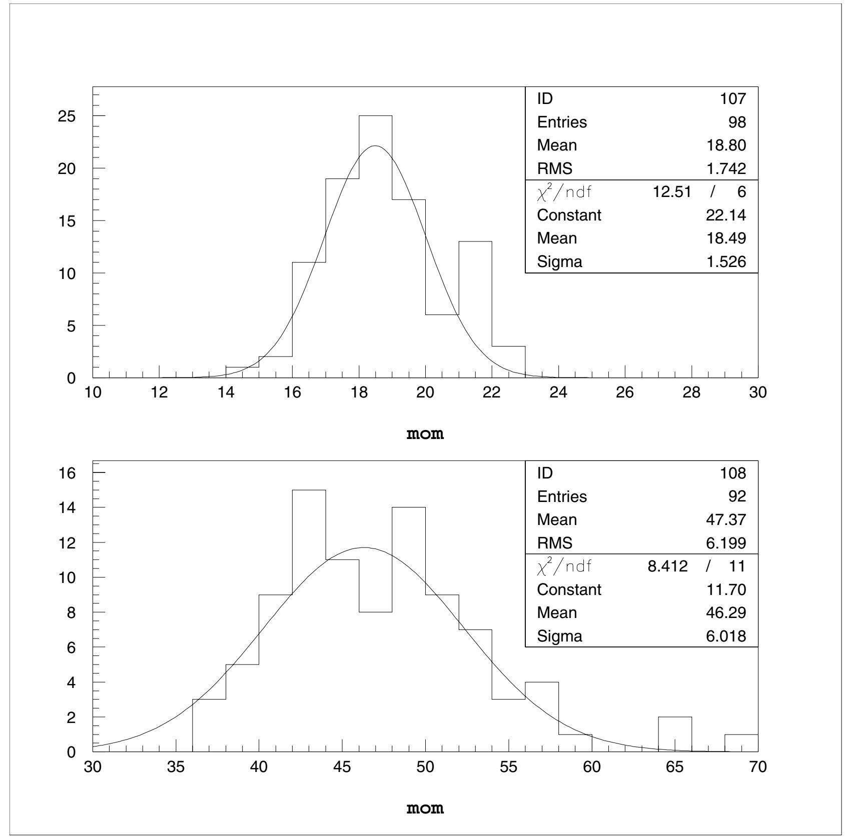

Figure 4. Reconstructed muon track momenta. The figure shows (*top*) 20 GeV/c and (*bottom*) 50 GeV/c.

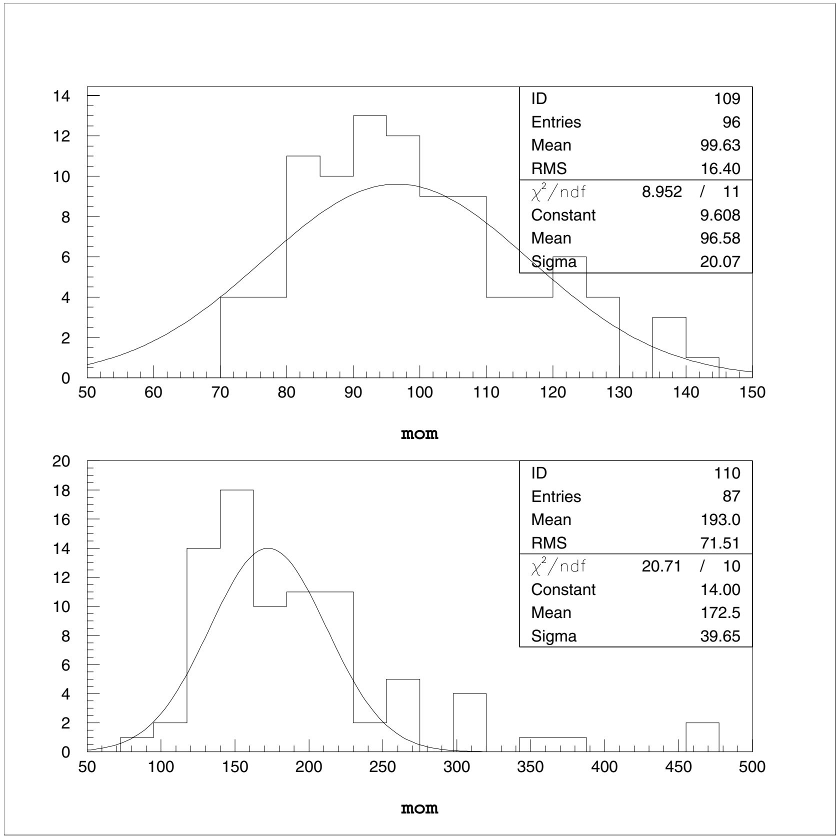

Figure 6. Reconstructed muon track momenta.
The figure shows (*top*) 100 GeV/*c* and (*bottom*) 200 GeV/*c*.

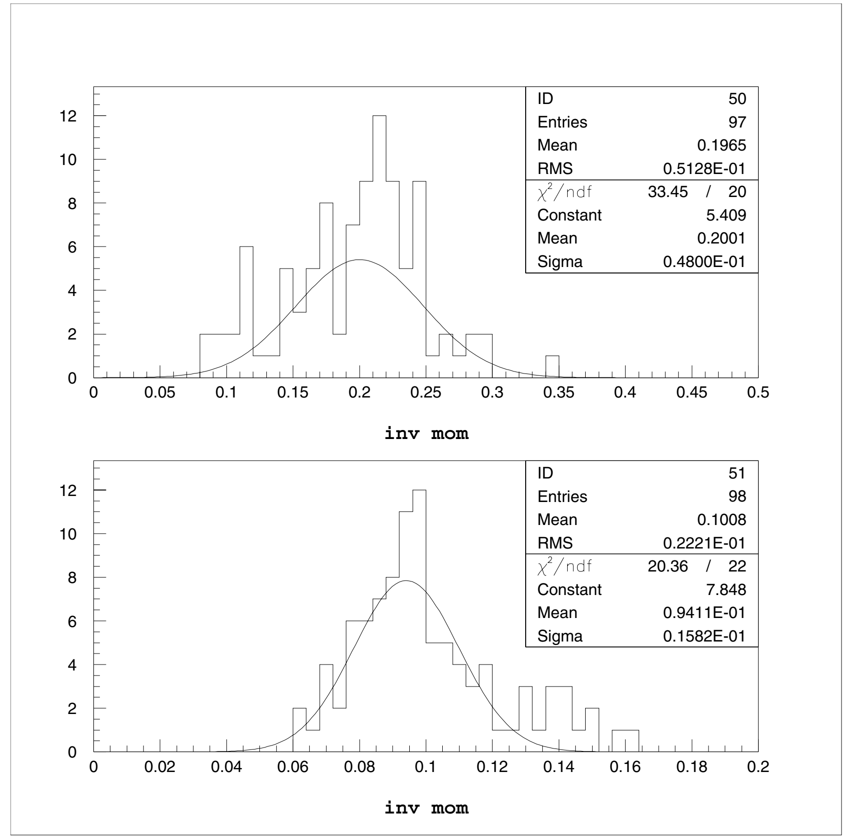

Figure 7. Reconstructed muon track 1/*p*.
The figure shows (*top*) 5 GeV/*c* and (*bottom*) 10 GeV/*c*.

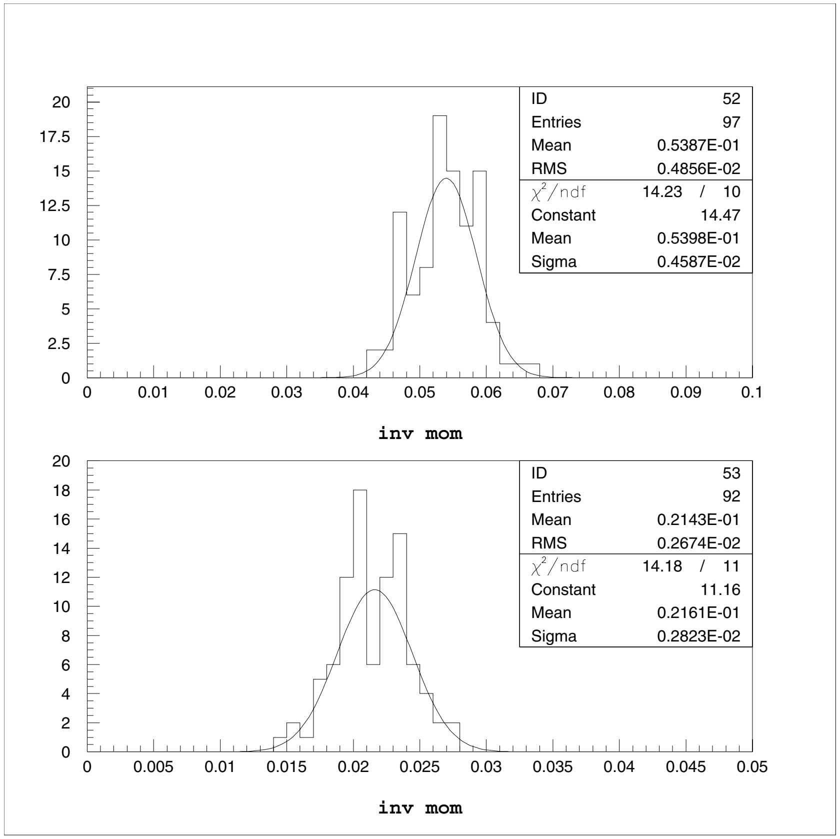

Figure 8. Reconstructed muon track 1/*p*.
The figure shows (*top*) 20 GeV/*c* and (*bottom*) 50 GeV/*c*.

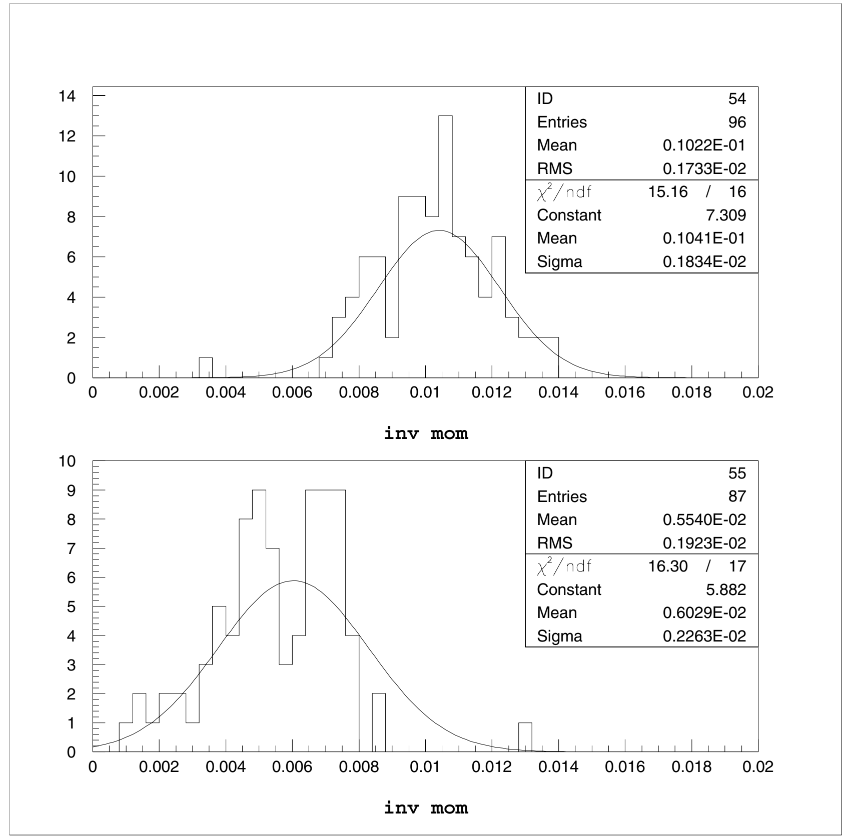

Figure 9. Reconstructed muon track 1/*p*.
The figure shows (*top*) 100 GeV/*c* and (*bottom*) 200 GeV/*c*.

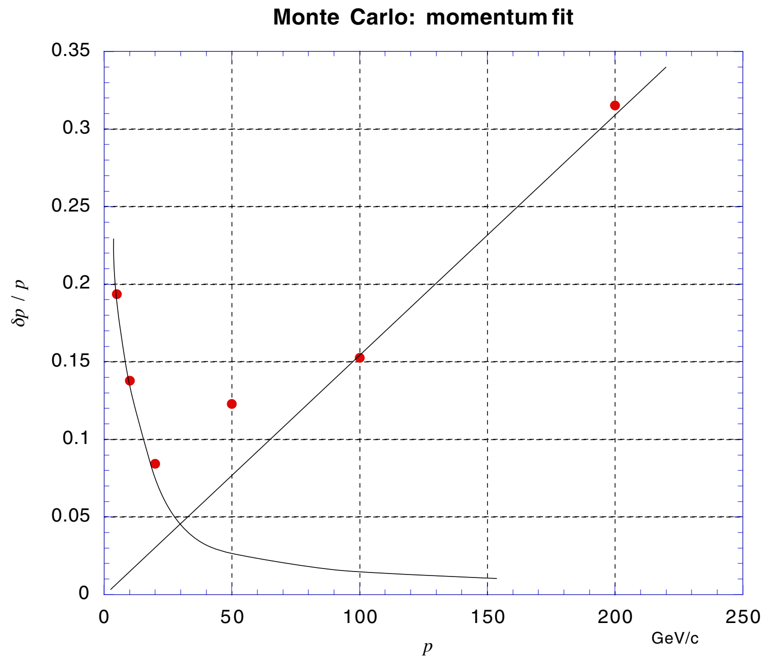

Figure 10. Relative error in momentum. Shown are the measured errors for momentum (dots) with the two main contributions: multiple scattering (like $1/p$) and measurement (like p). It must be noted that unlike errors in $(1/p)$, errrors in p are not symmetric about the mean, especially at large p.